Possible Existence Of Topological Excitations In Quantum Spin Models In Low Dimensions

Ranjan Chaudhury[a] and **Samir K. Paul**[b]

S. N. Bose National Centre For Basic Sciences

Block-JD, Sector-III, Salt Lake

Calcutta-700091, India

Abstract

The possibility of existence of topological excitations in the anisotropic quantum Heisenberg model in one and two spatial dimensions is studied using coherent state method. It is found that a part of the Wess-Zumino term contributes to the partition function, as a topological term for ferromagnets in the long wavelength limit in both one and two dimensions. In particular, the XY limit of the two-dimensional anisotropic ferromagnet is shown to retain the topological excitations, as expected from the quantum Kosterlitz-Thouless scenario.

a) **ranjan@boson.bose.res.in**

b) **smr@boson.bose.res.in**

Introduction

Quantum spin systems in low dimensions have aquired considerable significance in condensed matter Physics in recent times.In particular in two dimensional (2D) spin $\frac{1}{2}$ quantum Heisenberg antiferromagnet (QHAF) evoked a lot of interest in the light of discovery of high-temperature superconductors[1]. Many interesting theoretical and experimental work probing the magnetic property of various 2D systems and also that of many quasi-one dimensional (1D) systems brought into notice important features of anisotropic quantum spin models[2−4]. Parallel to this, possible extention of Kosterlitz-Thouless(KT) scenerio to quantum ferromagnetic spin models has also been attempted[5].

However, in spite of this endeavour many crucial questions have remained unanswered and in particular the origin of existence of topological excitations in quantum ferromagnetic and antiferromagnetic models seems to be mysterious . The existence of topological excitations in isotropic 1D-AF is well known[3,6].The case of 1D ferro (both isotropic and anisotropic) on the other hand has drawn lesser attention[3,6].One possible reason for this could be the lack of proper theoretical analysis of the quantum nature of the problem which we describe in this report. In the 2D case even for AF ,the issue of existence of topological excitations is still not settled fully although most of the theoretical calculations rule out such a possibility[6,7].Moreover, the high T_c oxides in the insulating antiferromagnetic (AF) phase seem to be governed by anisotropic (2D) Heisenberg model, whereas the theoretical efforts have mostly been confined to the isotropic case only[1,2,6,7].the 2D-ferro situation remained even less understood so far[5,6].

This motivated us to study the anisotropic quantum Heisenberg ferro and antiferromagnetic models in 1D and 2D,in a unified manner.

Mathematical Formulation

We analyse the quantum actions for XXZ ferromagnets and anti-ferromagnets in 1D and 2D by spin coherent state method[6].The philosophy behind this procedure is that,the existence of topological term in the full quantum partition functionof a quantum spin system implies topological excitations in the system[8]. Keepingin mind the physically relevant situations , we choose the anisotropy of the above spin models to be XY-like.

In the following we perform all the calculations on the lattice with a finite lattice parameter 'a' in the long wavelength limit.We write down the expressions of quantum Euclidean action in the quasicontinuum limit, so that we have a clear understanding of the topological terms while the physical system retains its lattice structure.

Calculations

The quantum Euclidean action $S_E[\mathbf{n}]$ for the spin coherent fields $\mathbf{n}(t)$ can be written as[6,9],

$$S_E[\mathbf{n}] = -isS_{WZ}[\mathbf{n}] + \frac{s\delta t}{4}\int_0^\beta dt\partial_t\mathbf{n}(t)^2 + \int_0^\beta dtH(\mathbf{n}) \tag{1}$$

where s is the magnitude of the spin and

$$H(\mathbf{n}) = \langle\mathbf{n}|H(\mathbf{s})|\mathbf{n}\rangle \tag{2}$$

$H(\mathbf{s})$ being the spin Hamiltonian in the representation s.The Wess-Zumino term S_{WZ} is given by[6]

$$S_{WZ}[\mathbf{n}] = \int_0^\beta \mathbf{dt} \int_0^1 \mathbf{d}\tau\mathbf{n}(\mathbf{t},\tau) \cdot \partial_\mathbf{t}\mathbf{n}(\mathbf{t},\tau) \wedge \partial_\tau\mathbf{n}(\mathbf{t},\tau) = \mathbf{A} \tag{3}$$

with $\mathbf{n}(t,0) \equiv \mathbf{n}(t)$,$\mathbf{n}(t,1) \equiv \mathbf{n_0}$, $\mathbf{n}(0,\tau) \equiv \mathbf{n}(\beta,\tau)$; $t \in [0,\beta]$, $\tau \in [0,1]$.

In (3) A is the area of the cap bounded by the trajectory Γ parametrized by $\mathbf{n}(t)$ on the sphere

$$\mathbf{n} \cdot \mathbf{n} = 1 \tag{4}$$

Here $|\mathbf{n}\rangle$ is the spin coherent state as defined in ref.(6).The spin Hamiltonian for XXZ Heisenberg ferromagnet is given by

$$H(\mathbf{S}) = -g\sum_{\langle\mathbf{r},\mathbf{r}\prime\rangle}\tilde{\mathbf{S}}(\mathbf{r}) \cdot \tilde{\mathbf{S}}(\mathbf{r}\prime) - g\lambda_z \sum_{\langle\mathbf{r},\mathbf{r}\prime\rangle}S_z(\mathbf{r})S_z(\mathbf{r}\prime) \tag{5}$$

with $g \geq 0$, $0 \leq \lambda_z \leq 1$, where $\mathbf{r}$ and $\mathbf{r}\prime$ run over the lattice and $\langle\mathbf{r},\mathbf{r}\prime\rangle$ signifies nearest neighbour interaction and $\mathbf{S} = (\tilde{\mathbf{S}}, S_z)$.

(i) **Linear Chain**

The quantum Euclidean action in the quasi-continuum limit can be written as :

$$\begin{aligned}S_E[\mathbf{n}] &= -isS_{WZ}^{NTOP} - \frac{is}{2}\int_{-L}^L dx\int_0^\beta dt\mathbf{n} \cdot \partial_\mathbf{t}\mathbf{n} \wedge \partial_\mathbf{x}\mathbf{n} + \frac{s\delta\mathbf{t}}{2\mathbf{a}}\int_{-L}^\mathbf{L}\mathbf{dx}\int_0^\beta\mathbf{dt}\\ &\quad(\partial_t\mathbf{n})^2 + \frac{s^2ga}{2}\int_{-L}^L dx\int_0^\beta dt[(\partial_x\tilde{\mathbf{n}})^2\\ &\quad+\lambda_z(\partial_xn_z)^2]\end{aligned} \tag{6}$$

where,$S_{WZ}^{NTOP} = 2\sum_W Z[\mathbf{n}(2ar)]$ is the nontopological part of the WZ termS_{WZ} on the chain, $L = 2ma$ and $\mathbf{n} = (\tilde{\mathbf{n}}, n_z)$. We have written down eq.(6) in the long wavelength limit.We analyse S_{WZ}term in the following manner :

$$\sum_\mathbf{r}S_WZ[\mathbf{n}(2ar)]$$

$$= \sum_{r=-m}^m S_WZ[\mathbf{n}(2ar)] + \sum_{r=-m}^{m-1} S_WZ[\mathbf{n}(2r+1)a]$$

$$\begin{aligned}
&= 2S_W Z[\mathbf{n}(-2am)] + S_W Z[\mathbf{n}(2am)] + 2\sum_{r=-m+1}^{m-1} S_W Z[\mathbf{n}(2ar)] \\
&\quad + \frac{1}{2}\int_{-L}^{L} dx \int_{0}^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} \\
&= 2S_W Z[\mathbf{n}(-2am)] + S_W Z[\mathbf{n}(2am)] + 2\sum_{r=-m}^{m-p} S_W Z[\mathbf{n}(2ar)] + 2[\int_{-L}^{L-(p-1)2a} dx \\
&\quad + \int_{-L}^{L-(p-2)2a} dx + \cdots \\
&\quad + \int_{-L}^{L-2a} dx]\int_{0}^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} \\
&\quad + \frac{1}{2}\int_{-L}^{L} dx \int_{0}^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} \tag{7}
\end{aligned}$$

Wher $1\leq p\leq 2m$. Since we keep the lattice parameter 'a' finite and we take $\delta t \to 0$,eq(6) takes the form :

$$S_E[\mathbf{n}] = -isS_{WZ}^{NTOP} - \frac{is}{2}\int_{-L}^{L} dx \int_{0}^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} + \frac{s^2 ga}{2}\int_{-L}^{L} dx \int_{0}^{\beta} dt[(\partial_x\tilde{\mathbf{n}})^2 + \lambda_z(\partial_x n_z)^2] \tag{8}$$

In order that the Euclidean action in (8) is finite for very large L we have :

$$\lim_{|\mathbf{x}|\to\infty}\partial_x\tilde{\mathbf{n}} = 0, \ \lim_{|\mathbf{x}|\to\infty}\partial_x n_z = 0\, i.e.\, \mathbf{n} \longrightarrow \mathbf{n}_0(t)\, on\, the\, circle\, x^2 + t^2 = R^2; R \to \infty\, with\, \mathbf{n}\cdot\mathbf{n} = 1 \tag{9}$$

This defines a mapping from (x,t)-space to the internal space $\mathbf{n}\cdot\mathbf{n} = 1$.However, there is a smaller class of $\mathbf{n}$ fields on the (x,t)-space which satisfies (9) with $\mathbf{n}_0(t)$ independent of t, denoted by $\mathbf{n}_0$. In that case the boundary points in the (x,t)-space are identified to a single point and we have a topological mapping $S_{Phys.}^2 \longrightarrow S_{Int.}^2$ with $\Pi_2(S^2) = Z$[10]. The winding number in this case is given by[8]

$$Q = \frac{1}{4\pi}\int dx dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} \tag{10}$$

where $Q \in z$.Thus for field configurations represented by

$$\{\mathbf{n}(x,t) : \lim_{|\mathbf{x}|\to\infty}\mathbf{n}(x,t) = \mathbf{n}_0\} \tag{11}$$

eqn(7) can be written as:

$$\begin{aligned}
\sum_{\mathbf{r}} S_{WZ}[\mathbf{n}(2ar)] &= 2\sum_{r=-m}^{m-p} S_{WZ}[\mathbf{n}(2ar)] + 2[\int_{-L}^{L-(p-1)2a} dx + \int_{-L}^{L-(p-2)2a} dx + \cdots \\
&\quad + \int_{L}^{L-2a}]\int_{0}^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} \\
&\quad + \frac{1}{2}\int_{-L}^{L} dx \int_{0}^{\beta} dt\, \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} \tag{12}
\end{aligned}$$

where we have made use of eq.(3).Notice that only the last term in eq.(12) covers the entire chain where the boundary points are identified via the configuration (11).Therefore the topological content in S_{WZ} is the last integral in (12) which is same as (10).The rest of

the term in (12) are non-topological.From eq.(7) the non-topological part can be written as:

$$S_{WZ}^{NTOP} = 2 \sum_{r=-m}^{m-1} S_{WZ}[\mathbf{n}(2ar)] \tag{13}$$

Due to translational invariance of $\mathbf{n}$ on the chain each term in (13) describes the same cap of area A given by eq.(3).Thus (13) can be written as:

$$S_{WZ}^{NTOP} = 2 \sum_{r=-m}^{m-1} A = 2(2m-1)A = constant \tag{14}$$

Therefore eq.(8) becomes:

$$S_E[\mathbf{n}] = -\frac{is}{2} \int_{-L}^{L} dx \int_0^\beta dt \mathbf{n} \cdot \partial_t \mathbf{n} \wedge \partial_x \mathbf{n} + \frac{s^2 ga}{2} \int_{-L}^{L} dx \int_0^\beta dt[(\partial_x \tilde{\mathbf{n}})^2 + \lambda_z(\partial_x n_z)^2] \tag{15}$$

The eq.(12) corresponding to an antiferromagnet ($g \leq 0$) reads

$$S_{WZ} = -\frac{1}{2} \int_{-L}^{L} dx \int_0^\beta dt \mathbf{n} \cdot \partial_t \mathbf{n} \wedge \partial_x \mathbf{n} \tag{16}$$

with S_{WZ}^{NTOP} vanishing due to staggering operation.So we get back the same result as obtained for isotropic antiferromagnet[6].

(ii) 2D Square Lattice
The spin Hamiltonian in this case is given by (5) where $\mathbf{r}$ runs over the 2D-square lattice.The quantum action for the anisotropic ferromagnet in the long wavelength limit is given by:

$$S_E[\mathbf{n}] = -is \sum_{\mathbf{n}} S_{WZ}[\mathbf{n}(\mathbf{r})] ++ \frac{gs^2}{2} \int_{-L}^{L} dxdy \int_0^\beta dt[(\partial_x \tilde{\mathbf{n}})^2 + \lambda_z(\partial_x n_z)^2 + (\partial_y \tilde{\mathbf{n}})^2 + \lambda_z(\partial_y n^z)^2](x,y,z) \tag{17}$$

Finiteness of the action (17) gives :

$$\mathbf{n}(x,y,t) \rightarrow \mathbf{n}_0(t) on the 2-sphere x^2 + y^2 + z^2 = R^2, R \rightarrow \infty and \mathbf{n} \cdot \mathbf{n} = 1 \tag{18}$$

This boundary condition gives a mapping from the (x,y,t)-space to the internal sphere $\mathbf{n} \cdot \mathbf{n} = 1$. However, (18) admits a class of $\mathbf{n}$ fields where $\mathbf{n}_0(t)$ is independent of t denoted by $\mathbf{n}_0$ in which case we have a mapping $S_{Phys}^3 \longrightarrow S_{Int}^2$ and $\Pi_3(S^2) = Z$[10]. Thus for the field configurations

$$\{\mathbf{n}(x,y,t) : \lim_{|x| \to \infty} \mathbf{n}(x,y,t) = \mathbf{n}_0\} \tag{19}$$

Following the same line of arguments as linear chain we can show that for field configurations satisfying (19) the S_{WZ} term corresponding to the 2D square lattice can be written as :

$$\sum_{\mathbf{r}} S_{WZ}[\mathbf{n}(\mathbf{r})] = S_{WZ}^{NTOP} + \frac{1}{2a} \int_{-L}^{L} dxdy \int_0^\beta dt[\mathbf{n} \cdot \partial_t \mathbf{n} \wedge \partial_x \mathbf{n} + \mathbf{n} \cdot \partial_t \mathbf{n} \wedge \partial_y \mathbf{n}](x,y,t) \tag{20}$$

$$S_{WZ}^{NTOP} = \{2(2m-1)\}^2 A \tag{21}$$

For the mapping $S_{Phys}^3 \longrightarrow S_{int}^2$, we can parametrize the (x,y,t)-space with boundary points identified by y-planes in which case $\frac{1}{4\Pi} \int_{-L}^{L} dx \int_0^\beta dt \mathbf{n} \cdot \partial_t \mathbf{n} \wedge \partial_x \mathbf{n}$ will be a winding

4

number through eq(10) for each y.This happens because of the fact that each (x,t)-plane (i.e $y = constant$) has its boundary points identified for field configurations satisfying (19) and the (x,t)-plane can be thought as a sphere S^2 passing through the north pole of S^3. So for each y we have a mapping from the corresponding (x,t)-plane ($= S^2$) to S^2_{Int} .Therefore we can write

$$\frac{1}{4\pi}\int_{-L}^{L}dxdy\int_{0}^{\beta}\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n}(x,y,t) = \int_{-L}^{L}dyQ(y) \tag{22}$$

By similar arguments :

$$\frac{1}{4\pi}\int_{-L}^{L}dxdy\int_{0}^{\beta}dt\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_y\mathbf{n}(x,y,t) = \int_{-L}^{L}dxQ(x) \tag{23}$$

Note that we can apply the above principle for the mapping $S^3_{Phys}\longrightarrow S^2_{Int}$ since $\Pi_3(S^2) = Z$[10].

Thus using eq(20),(21) the action (17) becomes :

$$
\begin{aligned}
S_E[\mathbf{n}] &= -\frac{is}{2a}\int_{-L}^{L}dxdy\int_{0}^{\beta}dt[\mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_x\mathbf{n} + \\
&\quad \mathbf{n}\cdot\partial_t\mathbf{n}\wedge\partial_y\mathbf{n}](x,y,t) + \frac{gs^2}{2}\int_{-L}^{L}dxdy\int_{0}^{\beta}dt \\
&\quad [(\partial_x\tilde{\mathbf{n}})^2 + \lambda_z(\partial_x n_z)^2 + (\partial_y\tilde{\mathbf{n}})^2 + \lambda_z(\partial_y n_z)^2](x,y,t)
\end{aligned} \tag{24}
$$

The first integral in (24) is a topological term as follows from (22) and (23). Let us point out that the right hand side of eq(20) vanishes identically on the long wave length limit under staggering operation in the case of antiferromagnets .

Conclusions

We have presented a unified scheme for analysing the topological terms in the effective action corresponding to the long wavelength limit of XY-like anisotropic quantum Heisenberg ferro and antiferro-magnets in one and two spatial dimensions for any value of the spin. Our calculation brings out clearly the hidden topological contribution from S_{WZ} term, which influences the statistical mechanics of ferromagnets in one dimension. This is probably manifested in the 'soliton-like excitations' occuring in many experimental systems corresponding to these models[3,4]. It may also be interesting to point out that in 1D the roles of kink and anti-kink are inter- changed as we go from ferro to anti-ferro due to sign reversal in the respective topological terms [see eqs. (15) and (16)]. In 2D situation in the limit $\lambda_z \to 0$, these excitations probably lead to the proposed "vortex-antivortex" scenario in the 'quantum KT' picture[5]. On the contrary the 2D-AF model does not exhibit any topological excitation in its long wavelength behaviour. Let us conclude by pointing out that our whole calculational approach is meaningful only in the low temperature regime where the spin-spin correlation length is appreciably large[6,11].

References

1

P.W. Anderson, Science, 235, 1196 (1987) ; P.W. Anderson, Phys. Rev. Lett., 59, 2407 (1987) ; J.G. Bednorz and K.A. Muller, Z. Phys., B64, 189 (1986) ; P. Chu et al, Phys. Rev. Lett., 58, 405 (1987).

2

Y. Endoh et al, Phys. Rev., B 37, 7443 (1988) ; K. Yamada et al, Phys. Rev., B 40, 4557 (1989) ; M. Sato et al, Phys. Rev. Lett., 61, 1317 (1988) ; R. Chaudhury, Ind. J. Phys. (Special issue on High Temperature Superconduc- tivity) 66A, 159 (1992).

3

G.M. Wysin and A.R. Bishop, Phys. Rev., B 34, 3377 (1986) ; H.J. Mikeska, J. Phys. C, 13, 2913 (1980) ; J.des Cloizeaux and J.J. Pearson, Phys. Rev., 128, 2131 (1967) ; Y. Endoh et al, Phys. Rev. Lett., 32, 170 (1974) ; M. Imada, 'Finite Temperature Excitations of the XYZ Spin Chain' (ISSP, Tokyo, 1982).

4

K. Hirakawa, H. Yoshizawa and K. Ubukoshi, J. Phys. Soc. Jpn., 51, 2151 (1982) ; K. Hirakawa et al, J. Phys. Soc. Jpn., 52, 4220 (1983) ; S. Komineas and N. Papanicolaou (Private Communications).

5

J.M. Kosterlitz and D.J. Thouless, J. Phys. C, 6, 1181 (1973) ; F.G. Mertens et al Phys. Rev. Lett., 59, 117 (1987) ; E. Loh, Jr., D.J. Scalapino and P.M. Grant, Phys. Rev., B31, 4712 (1985) ; F. Fucito and S. Solomon, DOE Research and Development Report, CALT-68-1023 (1981).

6

E. Fradkin and M. Stone, Phys. Rev., B 38, 7215 (1988) ; E. Fradkin, 'Field Theories of Condensed Matter Systems' (Addison Wesley, California, 1991).

7

P. Horsch in Lecture Note of the Mini-Workshop on ' Mechanisms for High Tempera- ture Superconductivity ' (ICTP, Trieste, 1988) ; S. Chakraverty, B.I. Halperin and D.R. Nelson, Phys. Rev. Lett., 60, 1057 (1988) ; S. Tyc, B.I. Halperin and S. Chakraverty, 62, 835 (1989) ; A. Auerbach and D.P. Arovas, Phys. Rev. Lett., 61, 617 (1988).

8

R. Rajaraman, 'Solitons and Instantons : An introduction to solitons and instantons in quantum field theory ' (North-Holland, Amsterdam, 1982).

9

A. Parola, Private Communications (1988).

10

N. Steenrod, 'The Topology of Fibre Bundles ' (Princeton University Press, New Jersey, 1951).

11

C.K. Majumdar in Proceedings of the Winter School and International Colloquium held at Panchgani, India, edited by B.S. Shastry, S.S. Jha and V. Singh (Springer-Verlag, Berlin Heidelberg, 1985), 142.